Dispersion, Roll and Aerodynamic Loading

Predictions

for

AUSROC II

Prepared Under Contract No.301481 for WSRL

by

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Summary

Estimates of flight dynamics and aerodynamics are used to provide information applicable to the flight safety of the AUSROC II rocket prior to firing from Woomera. Aerodynamic factors considered which affect range safety are flight path dispersion, rolling motion, interaction of body bending with aerodynamic loading, and aerodynamic load distribution on the fins. Predictions of flight path dispersion due to wind, fin misalignment and thrust misalignment are made. Rolling motion due to fin misalignment is estimated as a check for possible roll-yaw resonance. An approximation for the effect of body stiffness on the interaction between body bending and aerodynamic loading is derived. Simplified expressions for fin aerodynamic normal force distribution are presented to assist in checking fin structural strength.

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1. Introduction

AUSROC II is a bi-propellant liquid fuelled rocket designed by engineering students at Monash University as a final year project. This project has been reported in a Monash University, Department of Mechanical Engineering Final Year Project Thesis (1989) by Mark A. Blair and Peter Kantzos entitled "**Design of a Bi-Propellant Liquid Fuelled Rocket**". The rocket has a body diameter of 0.25m, a length of 5.56m and is capable of reaching a maximum altitude of around 20km. In order to test fly this rocket at Woomera there are safety requirements which have to be met.

The purpose of this Report, which has been prepared under contract for WSRL, is to provide information that will enable predictions to be made about the flight behaviour of AUSROC II as far as range safety is concerned. The information is derived from engineering estimates which allow reasonable checks on flight safety to be made when precise details of a rocket are not known. Flight safety is achieved by maintaining the rocket's flight path within safe boundaries over the firing range and by avoiding high angles of attack which could cause fracturing or break up of the rocket structure.

The main causes of deviation from the desired flight path are wind, deformation of the fins, and misalignment between motor thrust and the longitudinal axis of symmetry. Calculation of deviation arising from these three causes is carried out using methods taken from the book entitled "**Mathematical Theory of Rocket Flight**" by authors Rosser, Newton and Gross. Full details of these calculations are given in sections 2.1, 2.2, 2.3 and can be also used as examples of how to interpret the book.

An estimation of roll rate is needed to check that there is no danger of roll-yaw resonance leading to high angles of attack. This estimate is made in section 3 together with initial roll history.

Another check which needs to be made is that the rocket body is stiff enough to resist large magnification of aerodynamic loading due to body bending. A criterion which relates this load magnification to body stiffness is derived in section 4.

To enable analysis of fin structural strength to be carried out, a simplified procedure for calculating the normal aerodynamic force distribution on a fin at maximum dynamic pressure is derived in section 5.

A summary of the results of the calculations carried out in this report is given in section 6.

2. Dispersion

The dispersion of a rocket is regarded as its deviation from some ideal flight path due to such causes as wind, imperfections in manufacture, and launch disturbances. Estimates of dispersion of the AUSROC rocket during boosted flight have been made using the methods and tabulated functions given in the well-known book entitled "**Mathematical Theory of Rocket Flight**" by authors Rosser, Newton and Gross, published by Mc Graw-Hill in 1947 (Ref.1).

For AUSROC II, the ideal flight path is taken to be the computed particle trajectory which takes account of aerodynamic drag and gravity. In reference (1) the equations of motion for the rocket are simplified so that they can be solved analytically for angular deviation and displacement of the dispersed flight path from the ideal one. These simplified differential equations are linear and hence their particular solutions are additive so that the combined effect of all causes of dispersion is the vector sum of the effects from each individual cause. The simplifying assumptions are that aerodynamic and gravitational forces can be ignored, leaving motor thrust as the sole motivating force which, together with rocket acceleration, is assumed to be constant. The only aerodynamics considered is the pitching or yawing moment which is assumed to have a constant aerodynamic coefficient. These assumptions may appear to be pretty drastic but it should be remembered that they need only be used to estimate perturbations about the ideal trajectory and not to estimate the ideal trajectory itself. In practice, most of the dispersion of an accelerating rocket, launched from rest, occurs in a relatively short part of the flight path immediately after leaving the launcher. Thus, taking the required constant values for the various rocket properties to be the launch values, will usually result in good engineering estimates for dispersion.

A consequence of these assumptions is that the response of the rocket to a pitching disturbance is to perform simple harmonic oscillations such that the wavelength with respect to flight path distance is constant. For a given rocket, this wavelength is used to non-dimensionalise flight path distance and launcher length. The analytic functions in reference (1) are given in terms of these two non-dimensional variables.

It is obviously very important not to make errors in determining the direction of the various components of dispersion. Thus we need to define some systems of axes and to adopt a sign convention with regard to angles in a way that assures compatibility with the formulae quoted in reference(1). The ideal trajectory for the rocket during boost, as calculated by a particle trajectory computer program, is contained in a vertical plane. In this plane, the positive x-axis is defined as the horizontal axis passing downrange from the launcher. Looking downrange, the positive y-axis passes horizontally outwards to the left from the launcher and is at right angles to the x-axis. The positive z-axis passes vertically upwards from the launcher. Thus this x, y, z system of axes, with origin at the launcher, is a fixed range system and forms a cyclic right-handed axes set. Note that in explaining the treatment of dispersion due to crosswind, reference (1) describes these axes in the order x,z,y,but there need be no confusion because these coordinates are not used in the dispersion formulae. Defining the range-fixed unit vectors **i**, **j**, **k**, in the positive x, y, z directions respectively, we have the standard cyclic order that gives:

$$\mathbf{i} \times \mathbf{j} = \mathbf{k} \qquad \mathbf{j} \times \mathbf{k} = \mathbf{I} \qquad \mathbf{k} \times \mathbf{i} = \mathbf{j}$$

Another axes system, which varies from point to point along the ideal trajectory, will now be introduced. Take **t** to be the unit vector tangential to the flight path at any point on the ideal trajectory and positive in the direction of flight. Let **n** be the unit vector normal to **t**, lying in the vertical plane containing the ideal trajectory and taken to be positive when it has an upward pointing component. Again **j** is the unit vector normal to **t** and **n** ; taken as positive to the left of the flight path when looking downrange. Hence, the vectors **t** , **j** , **n** , form a cyclic right-handed system similar to **i** , **j** , **k** .

ie: **t** x **j** **=n** **j** x **n** **=t** **n** x **t** **=j .**

The planes in which dispersion are measured can now be defined. Angles of dispersion are deviations of the dispersed trajectory from the **t** direction, which is tangential to the ideal trajectory. These angular deviations can occur in any direction and accordingly are treated as vectors having two components at right angles. This treatment does not normally hold for angular displacements, but is a close approximation when the displacements are small angles of the first order: ie when $\mathrm{Sin}\theta=\theta$ and $\mathrm{Cos}\theta=1$ are sufficiently accurate approximations. Thus an angular deviation θ from the **t** direction is assumed to have two components, a component θ_V in the vertical **t** , n plane and a component θ_S in the side **t** , **j** plane. Thus, the angular deviation about **t** can be considered as the complex variable $\theta v + i\theta_S$. Note that in reference (1) angular deviation θ is a complex number but the subscripts v and s are not used. The sign conventions for θ_V and θs are that θv is positive for an upwards deviation from **t** and negative for a downwards deviation from **t** . θ_S is positive for a leftward sideways deviation from **t** and negative for a rightward sideways deviation from **t** , looking downrange in both cases.

The definitions above are in accord with the dispersion formulae of reference (1) and we are now in a position to carry out dispersion calculations without any ambiguity concerning the direction of trajectory deviations.

The constants needed for making the calculations described in reference (1) will now be determined for AUSROC. These are, the initial acceleration G, the effective launcher length p, the launch velocity V_p and the wavelength in yaw σ.

Initial thrust T = 9810 N,

Initial mass m = 212 kg,

$$\text{then } G = T / m = 46.27 \text{ m} / \text{s}^2 \qquad \textbf{(1)}$$

The effective launcher length p is defined as the distance travelled by the centre of gravity of the rocket from rest until the second lug is just clear of the launcher rail. The value taken for AUSROC is:

$$p = 9.7 \text{ m}.$$

The launch velocity Vp is then found by putting:

$$V_p^2 = 2\,G\,p$$

So that: $V_p = 29.3 \text{ m/s}^2$

From reference (1), the wavelength in yaw or pitch, σ, is found from

$$\sigma^2 = \frac{4\,I\,\pi^2}{\rho\, d^3\, K_m} \qquad \textbf{(2)}$$

where: $I = 268 \text{ kgm}^2$ (initial pitching moment of inertia)
$\rho = 1.23 \text{ kg/m}^3$ (air density at sea level)
$d = 0.25$ m (rocket body diameter)

K_m is an aerodynamic pitching moment coefficient such that the aerodynamic pitching moment about the rocket's centre of gravity is given by:

$$M = \rho\, V^2\, d^3\, K_m\, \alpha$$

where α is the angle between the longitudinal axis of the rocket and the relative wind direction. The more usual form, used in wind tunnel testing is:

$$M = 0.125\, \rho\, V^2\, \pi\, d^3\, (dC_m / d\alpha)\, \alpha$$

$$K_m = 0.125\, \pi\, (dC_m / d\alpha)$$

The Aerodynamic Prediction Program of Guided Weapons Division predicts, for AUSROC at subsonic speed:

$$dC_m / d\alpha = 89$$

hence: $K_m = 35$

Substituting the above values into equation **(2)** gives:

$$\sigma = 125.4 \text{ m} \qquad \textbf{(3)}$$

In reference (1), distance travelled along the flight path by the rocket's centre of gravity is denoted by s and the non-dimensional quantities related to effective launcher length p and s are denoted by S and P respectively where:

$$S = 2\,\pi\, s\ /\ \sigma = s\ /\ 20.0 \qquad \textbf{(4)}$$

$$P = 2\,\pi\, p\ /\ \sigma = 0.465 \qquad \textbf{(5)}$$

The rocket functions from which dispersion is calculated, are expressed in terms of S and P in reference (1). Thus, to calculate dispersion at the end of burning an appropriate value for S is required. Subscript b is used to denote conditions at motor burnout, and subscript p denotes conditions when the rocket just becomes clear of the launcher.

An approximate value for s_b is found by assuming the motion during burning to have a constant acceleration equal to the mean value. From the particle trajectory, the rocket velocity at motor burnout, which is at 20 seconds, is given by V_b = 749 m/s so that the mean acceleration is 37.45 m/s^2, thus:

$$s_b = 0.5 \times 37.45 \times 20^2 = 7490 \text{ m}$$

$$S_b = 7490 \;/\; 20.0 = 374.5 \qquad \textbf{(6)}$$

Angular dispersion is relatively insensitive to values of s greater than one wavelength in yaw, as the rocket functions evaluated in reference (1) will show. The value of s_b for AUSROC is equivalent to about 60 wavelengths and will have a very small influence on the calculated values of angular dispersion. Numerically, the dominant effect will arise from P. Having established numerical values for P and S_b, the values of the functions needed to calculate dispersion at the end of burning will now be determined from reference (1). Using the notation of reference (1), the functions to be evaluated are G_1, G_2, G_3 & G_4 and these are given in terms of the tabulated rocket functions ir, ra^2, rr, rj. Because S is large, the asymptotic expansions for the G functions and their integrals can be used and are given in equations 111.1.62 to 111.1.69 of reference (1). In Table 1 of reference (1), the rocket functions are tabulated for values of their arguments ranging from 0 to 51. Series expansions for small and large arguments are given in chapter V. Plots of G(S,P) for 0<S<14 and 0<P<1.5 are shown in chapter 111. They are useful for checking calculated values and for observing trends.

From equations 111.1.62 to 111.1.69, for large S:

$$G_1(S,P) = 0.5 \{ ir(P) - ir(S) \} \qquad \textbf{(7)}$$
$$G_2(S,P) = 0.25 \{ ra^2(P) - (1/s) \} \qquad \textbf{(8)}$$
$$G_3(S,P) = 1 - vP\, rr(P) \qquad \textbf{(9)}$$
$$G_4(S,P) = vP\, rj(P) \qquad \textbf{(10)}$$

For S larger than the maximum tabulated value of 51, put:

$$ir(S) = 1.96351 + \ln(S)$$

The integrals of G are found from:

$$\int_P^S G_1\, dS = SG_1 - 0.5G_2 + 0.5(S - P) \qquad \textbf{(11)}$$

$$\int_P^S G_2\, dS = SG_2 + 0.5G_1 \qquad \textbf{(12)}$$

$$\int_P^S G_3\, dS = SG_3 - 0.5G_4 \qquad \textbf{(13)}$$

$$_P\int^S G_4\, dS = SG_4 + 0.5G_3 - 0.5 \qquad \textbf{(14)}$$

The rocket functions to be evaluated from Table 1 of reference (1) are thus ir(P), ra^2(P), rr(P), rj(P), for P = 0.465. Interpolation between the tabulated values gives:

ir(0.465) = 1.5541 $\quad$ ra^2(0.465) = 1.0991

rr(0.465) = 0.9707 $\quad$ rj(0.465) = 0.3960

Substituting these values into equations (7) to (14) gives:

$$G_1(Sb) = -3.1675 \quad \therefore \quad _P\int^S G_1\, dS = -999.3$$

$$G_2(Sb) = 0.2741 \quad \therefore \quad _P\int^S G_2\, dS = 101.1$$

$$G_3(Sb) = 0.3381 \quad \therefore \quad _P\int^S G_3\, dS = 126.4$$

$$G_4(Sb) = 0.2701 \quad \therefore \quad _P\int^S G_4\, dS = 100.8$$

These values can now be inserted into the dispersion formulae, equations 111.1.6 and 111.1.43 of reference (1) to calculate individual components. The effect of a steady cross wind will now be considered.

2.1 Dispersion Due to Steady Cross Wind

Dispersion arising from a steady atmospheric wind is not treated directly as such in reference (1), but can be made to conform to a direct treatment by considering motion relative to an axes frame that moves with the atmosphere. This transforms the problem into that of a rocket launched into still air at some initial angle of attack δ_p and with an initial velocity which is not parallel to the launcher rail.

We start by considering a rocket which is being launched into a steady cross wind and which is moving parallel to, and along, the rail of its launcher. Let the launcher point down range in the **i**, **k** plane and let the cross wind vector be **j**v, where v is a positive constant. Thus, according to our sign convention, the cross wind is horizontal and is blowing from right to left looking down range from the launcher. Now we have a dynamic system consisting of three elements, a launcher at rest, a rocket moving with launch velocity V_p**t**, and an atmosphere moving with constant velocity **j**v. If we superimpose a velocity -**j**v upon this system we will end up with an atmosphere at rest, a launcher moving with velocity -**j**v and a rocket with launch velocity V_p**t** - **j**v.

Dispersion arising from these conditions can be calculated directly from reference (1). This dispersion, so calculated, gives the displacement of the rocket relative to the moving atmosphere, but we want displacement relative to the earth. By simply adding **j**vt to the displacement vector relative to the atmosphere we obtain displacement relative to earth, where t is motor burning time after the rocket leaves the launcher.

A mathematical justification for the foregoing procedure can be made as follows. In reference (1), the dispersion of a rocket launched into still air at some angle of attack and with a velocity not aligned with the launcher is found from solutions to the equation of motion:

$$m\,(d\mathbf{V}/dt) = \mathbf{T}(\,\mathbf{V_p}, s\,) \qquad \textbf{(15)}$$

together with a simple aerodynamic pitching moment equation which relates the direction of thrust vector **T** to flight path distance s by a simple harmonic oscillation about the $\mathbf{V_p}$ direction and is independent of V. More generally, aerodynamic moments acting on a rocket are dependent on the velocity relative to the air, $\mathbf{V_{RA}}$, and only on **V** when the air is at rest. Thus, for air moving with velocity $\mathbf{V_w}$, equation (15) becomes:

$$m\,(d\mathbf{V}/dt) = \mathbf{T}(\,\mathbf{V_{RAp}}, s\,) \qquad \textbf{(16)}$$

where $\mathbf{V_{RAp} = V_p - V_w}$

and $\mathbf{V_{RA}} = \mathbf{V} - \mathbf{V_w}$ **(17)**

The assumptions here are that the magnitude of the thrust T and the pitching frequency remain unchanged when the magnitude of the rocket’s velocity is changed to its value relative to the moving air.

When $\mathbf{V_w}$ is constant, it can be seen from equation (17) that:

$$(d\mathbf{V}/dt) = (d\mathbf{V_{RA}}/dt)$$

which, when substituted into equation (16) gives:

$$(d\mathbf{V_{RA}}/dt) = \mathbf{T}\,(\mathbf{V_{RAp}}, s) \qquad \textbf{(18)}$$

This equation is of the same form as equation (15) and therefore has the same general solution in which different initial conditions are inserted. Having thus found the displacement for $\mathbf{V_{RA}}$, relative to the moving air, it can be seen from equation (17) that displacement relative to earth is found by adding $\mathbf{V_w}$ t when $\mathbf{V_w}$ is expressed in terms of earth fixed unit vectors (eg **i** , **j** , **k**) and t is time of flight.

Now return to the calculation of dispersion due to a cross wind **j**v. Relative to the cross wind, the rocket leaves the launcher with velocity:

$$\mathbf{V_{RAp}} = \mathbf{V_p} - \mathbf{j}v = V_p\mathbf{t} - \mathbf{j}v$$

and the rocket’s longitudinal axis is aligned with the launcher in the **t** direction. Thus $\mathbf{V_{RAp}}$ lies in the **t**, **j** side plane and, within the assumption of first order deviation angles, makes an angle of $-v/V_p$ with the ideal trajectory direction **t**. Now use the solutions given in reference (1) for a rocket launched into still air, with an initial velocity deviation from the ideal trajectory, $-v/V_p$, at initial angle of attack $\delta_p = v/V_p$.

The angle of attack in the moving axes frame is that which would occur for a rocket with velocity $V_p\mathbf{t} - \mathbf{j}v$ launched into still air and with its longitudinal axis lying along the **t** direction. In the **t**, **j** side plane, the angle of attack is positive when the nose of the rocket lies to the left of **t** looking down range. Hence, in the moving axes frame:

$$\delta_p = (v/V_p) \text{ (to first order)}$$
$$\theta_{hp} = -(v/V_p)$$

Strictly, these angles, being in the side **t**, **j** plane, are the imaginary components of the complex number representation of δ_p and θ_p used in the formulae of reference (1). Thus, for substitution into these formulae, put:

$$\delta_p = i\,(v/V_p)$$
$$\theta_{hp} = -i\,(v/V_p)$$

From reference (1), equation 111.1.43 gives:

$$\eta = (\sigma/2\pi)\,(\theta_p - \theta_a)\,(S - P) + (\sigma/2\pi)\,{}_P\!\int^S G_3\,dS$$

where η is the displacement of the rocket from the ideal flight path at distance S.θ_a is the angle between the ideal flight path and the appropriate reference axis at launch. Taking this reference direction as **t** for the cross wind dispersion makes θ_a equal to zero in the side plane. Thus we have:

$$\eta = -i(v/V_p)\,(\sigma/2\pi)\,(S - P) + i(v/V_p)\,(\sigma/2\pi)\,{}_P\!\int^S G_3\,dS$$

This gives η relative to the cross wind, so for η relative to earth we must add ivt where t is time of flight. Finally, we get:

$$\eta = -i(v/V_p)\,(\sigma/2\pi)\,(S - P) + i(v/V_p)\,(\sigma/2\pi)\,{}_P\!\int^S G_3\,dS + ivt \qquad \textbf{(19)}$$

Differentiating equation (19) with respect to s gives angular dispersion:

$$\theta = -i(v/V_p) + i(v/V_p)\,G_3 + i(v/V) \qquad \textbf{(20)}$$

At the end of burning:

$S_b = 374.5$
$t = 20s$
$P = 0.465$
$\sigma = 125.4m$
${}_P\!\int^S G_3\,dS = 126.4$

Thus, from equation (19):

$$\eta_b = -i(v/V_p)\,7465 + i(v/V_p)\,2523 + iv\,20$$

and for $V_p = 29.3$ m/s:

$$\eta_b = -\mathbf{i}\ 149\ v\ \text{m}.$$

This means that the trajectory displacement from the ideal at all burnt is 149 metres in the -i direction per unit of cross wind v. From our sign convention, the -i direction is horizontally outward to the right, looking down range from the launcher. This movement is in response to a cross wind blowing from the right.

From equation (20), at all burnt, with: $G_3(S_b, P) = 0.3381$
$V_b = 750$ m/s

$$\theta_b = -i(v/V_p) + i(v/V_p)\,0.3381 + i(v/750)$$

$$= -iv\,0.0213 \text{ for } V_p = 29.3 \text{ m/s} \qquad \textbf{(21)}$$

Thus θ_b is in the negative part of the **t** ,**j** plane, which means that the dispersed flight path is displaced to the right of **t** looking down range. To find the horizontal projection of this angle in the **i** ,**j** plane, divide by the cosine of the angle between the ideal flight path direction **t** and the horizontal down range direction **i**. From the computed particle trajectory, this angle is 46 degrees at all burnt.

Hence flight path displacement angle measured from **i** in the horizontal ground plane is:

$$= -iv\,(0.0213/\cos 46°)$$
$$= -iv \times 0.0307 \text{ radian}$$
$$= -iv \times 1.76 \text{ degrees} \qquad \textbf{(22)}$$

This means that for each m/s of cross wind v, blowing from right to left, looking down range, the projection of the trajectory in the horizontal ground plane at all burnt will have deviated 1.76 degrees to the right of the horizontal direction of fire.

2.2 Dispersion Due to Steady Tail Wind

The significance of a tail wind is that it increases the effective launcher elevation and it is important to avoid effective launch angles approaching 90 degrees. Thus an estimate of angular dispersion in the vertical plane is required. We proceed in the same manner as for the cross wind, but now the **t**, **n** axes in the vertical plane through the ideal trajectory are the reference directions. If θ_a is the angle between the tangent to the ideal trajectory and the horizontal (ie the angle between **t** and **i**) then the axes transformation equations are:

$$\mathbf{i} = \mathbf{t}\,\mathrm{Cos}\,\theta_a - \mathbf{n}\,\mathrm{Sin}\,\theta_a$$

$$k = \mathbf{t}\,\mathrm{Sin}\,\theta_a + \mathbf{n}\,\mathrm{Cos}\,\theta_a$$

in the vertical plane.

Let the tail wind vector be **i**u, where u is a positive constant. Then:

$$\mathbf{V}_{RA} = \mathbf{V} - \mathbf{i}u = Vt - u(\mathbf{t}\,\text{Cos}\,\theta_a - \mathbf{n}\,\text{Sin}\,\theta_a)$$
$$= \mathbf{t}\,(V - u\,\text{Cos}\,\theta_a) + \mathbf{n}u\,\text{Sin}\,\theta_a$$

Ignoring u Cos θ_a in comparison with V, the relative velocity vector at launch is given by:

$$\mathbf{V}_{RAp} = \mathbf{t}\,V_p + \mathbf{n}u\,\text{Cos}\,\theta_{ap}$$

where **t** and **n** are the unit tangential and upward normal direction vectors respectively of the ideal trajectory at launch. Thus $\mathbf{V}_{RAp}$ lies above **t** in the positive part of the **t**, **n** plane and the deviation angle:

$$(u\,/\,V_p)\,\text{Cos}\,\theta_{ap} \qquad \text{is thus positive.}$$

The angle of attack in the moving axes frame is that which would occur on a rocket with velocity $V_p\mathbf{t} + \mathbf{n}\,u\text{Sin}\theta$ launched into still air with its longitudinal axis lying along the **t** direction. Thus the nose of the rocket lies below **t** in the negative part of the plane and hence the angle of attack at launch:

$$\delta_p = -(u\,/\,V_p)\,\text{Sin}\theta$$

Hence, in the moving axes frame:

$$\delta_p = -(u\,/\,V_p)\,\text{Sin}\theta_p$$
$$\theta_{vp} = (u\,/\,V_p)\,\text{Sin}\theta_p$$

When substituted into equations (19) and (20) the numerical results for dispersion will be the same as for the cross wind but with iv replaced by -u Sin $\theta_{p.}$

Thus, from equation (21) $\quad \theta_b = u\,\text{Sin}\theta_{ap} \times 0.0213$

$$= 0.0200\,u \text{ for } \theta_{ap} = 70 \text{ degrees}$$

This means that for every m/s of tail wind the launch angle is effectively increased by 0.0200 radians or 1.147 degrees. Thus a tail wind of 17.4 m/s would raise the effective launcher angle to 90 degrees from the actual value of 70 degrees.

This is a conservative result because it adds the total angular deviation at all burnt to the launcher angle to get the effective angle of launch. What actually happens is that the angular dispersion starts off at zero at the launcher and as it builds up to its maximum value the ideal trajectory angle decreases. This behaviour could be calculated for a given value of tail wind if considered critical in application to a particular firing.

2.3 Dispersion Due to Fin Misalignment

When the fins are misaligned with the body or are distorted they generate an aerodynamic lifting force while the body is at zero angle of attack. In free flight, under these conditions, the body will oscillate about the angle of attack at which the pitching moment about the centre of gravity is zero. The thrust is then off set from the direction of motion along the ideal trajectory, causing dispersion. When a rocket rolls during flight, the off set thrust vector rotates and dispersion is reduced. Misaligned or distorted fins are very likely to induce rolling motion. In order to use the simplest results of reference (1), the assumption that there is no rolling motion will be made. The calculation thus results in a maximum possible value and serves to indicate the sensitivity of dispersion to fin misalignment.

Fin misalignment can be represented by a fin deflection angle relative to the body. For the AUSROC four fin configuration, fin misalignment can be represented by an angle ε, defined as the deflection angle that two adjacent fins would possess to generate the same aerodynamic moment as the actual misalignment or distortion.

The aerodynamic moment about the centre of gravity, due to fin misalignment is expressed as:

$$M = Q\, S\, d\, \varepsilon\, (dCm_f / d\alpha)$$

where: Q = dynamic pressure (Pa)

S = reference area (m^2)

d = body diameter (m)

$dCm_f / d\alpha$ = aerodynamic pitching moment coefficient derivative for the fins alone.

When the body sits at angle of attack δ_m, the corresponding aerodynamic moment is:

$$M = Q\, S\, d\, \delta_m\, (dCm / d\alpha)$$

where: $dCm / d\alpha$ = aerodynamic pitching moment coefficient derivative for the complete rocket with no fin distortion.

Equating these two moments, gives:

$$\varepsilon\, (dCm_f / d\alpha) = \delta_m\, (dCm / d\alpha)$$

Most of the dispersion occurs just after launch, so the aerodynamic moment coefficient derivatives are taken as their subsonic values from the Aerodynamic Prediction Program. These are:

$$dCm_f / d\alpha = -1.9865 \ / \ \text{degree}$$
$$dCm / d\alpha = -1.5534 \ / \ \text{degree}$$

Thus: $\delta_m = 1.279\ \varepsilon$

Equations 111.1.6 and 111.1.43 of reference (1) give the angular deviation θ and displacement η due to δ_m as:

$$\theta = -\delta_m (G_1 + G_3)$$
$$\eta = -(\sigma/2\pi)\ \delta_m\ {}_P\!\int^{S} (G_1 + G_3)\ dS$$

Thus for:

$$G_1(S_b) = -3.1675$$
$$G_3(S_b) = 0.3381$$

$${}_P\!\int^{Sb} G_1\ dS = -999.3$$
$${}_P\!\int^{Sb} G_3\ dS = 126.4 \text{ (as tabulated in section 2)}$$

$$\theta_b = 3.62\ \varepsilon$$

and $\eta_b = 22282\ \varepsilon$ (ε in radians)

It follows that for a non-rolling rocket where fin misalignment is such that the thrust offset is restricted to the side **t**, **j** plane, the flight path deviation in this plane at all burnt would be i3.62 ε from the **t** direction. When projected into the horizontal plane, the equivalent flight path deviation at all burnt is:

$$i\ 3.62\ \varepsilon \ / \ \text{Cos } 46° = i\ 5.2\ \varepsilon \qquad \textbf{(23)}$$

from the horizontal down range i direction.

2.4 Dispersion due to thrust misalignment

When the thrust vector does not pass through the centre of gravity of the rocket a pitching moment is set up. This moment is countered by the aerodynamic restoring moment on a stable rocket. Such a rocket will oscillate about the angle of attack at which the aerodynamic moment balances the thrust moment. Hence the thrust will have a component normal to the ideal flight path and this will cause dispersion. Using the notation of reference (1), the thrust vector is assumed to make an angle δ_T with the axis of the rocket and to have a moment arm L about the centre of gravity. δ_T is taken as positive in the vertical plane when the resulting pitching moment tends to move the nose of the rocket up above the flight path. The side plane component of δ_T is considered positive when the resulting pitching moment tends to move the nose of the rocket to the left of the flight path, looking down range. Side plane components of δ_T are prefixed by i because δ_T is considered as a complex number in reference (1).

Equations 111.1.6 and 111.1.43 of reference(1) give the angular dispersion θ and displacement η due to thrust misalignment as:

$$\theta = \delta_T G_1 + (\sigma L / 2\pi k^2) G_2$$

$$\eta = (\sigma/2\pi) \delta_T \int_P^S G_1 \, dS + (\sigma^2 L / 4\pi^2 k^2) \int_P^S G_2 \, dS$$

where k is rocket radius of gyration in pitch = 1.11m

Thus, for: $G_1(S_b) = -3.1675$ $G_2(S_b) = 0.2741$

$\int_P^{Sb} G_1 \, dS = -999.3$ $\int_P^{Sb} G_2 \, dS = 101.1$

(as tabulated in section 2)

To determine L, it is assumed that the distance of the centre of gravity from the motor nozzle is 2m for AUSROC. Thus for L=2 δ_T:

$$(\sigma L / 2\pi k^2) = 32.4 \delta_T$$

$$\theta_b = 5.71 \delta_T$$

$$\eta_b = 45403 \delta_T$$

Thus if the misalignment is restricted to the **t**, **j** side plane:

$$\theta_b = i \, 5.71 \delta_T \qquad \textbf{(24)}$$

and when projected into the horizontal plane the equivalent flight path deviation at all burnt is:

$$i \, (5.71 \delta_T \; / \; \text{Cos } 46°) = i \, 8.22 \delta_T$$

from the horizontal down range i direction.

2.5 Range safety calculation

Take an arc of 45 degrees in the horizontal ground plane down range of the launcher as the boundary over which the rocket must not fly. This leaves 22.5 degrees on either side of the down range horizontal axis direction as the maximum side dispersion. The worst case of side dispersion that could occur is the unlikely one where all the effects of fin misalignment, thrust misalignment, and cross wind are all directed towards the same side direction. At all burnt, the ideal trajectory (ie the computed particle trajectory) gives the horizontal down range distance as 5000m and the flight path angle as 46 degrees above the horizontal. Hence the maximum allowable side displacement at all burnt is:

$$5000 \operatorname{Tan}22.5° = 2071 \text{ m}$$

Taking the results for the components of η_b and θ_b from sections 2.4, 2.3 and 2.1 the conditions to be satisfied by v, ε and δ_T are:

$$149v + 22282\,\varepsilon + 45403\,\delta_T < 2071 \qquad \textbf{(25)}$$

$$(1/\operatorname{Cos}46°)\,(0.0213v + 3.62\,\varepsilon + 5.71\,\delta_T) < 22.5 \times \pi\,/180$$

$$\text{ie. } 0.0307v + 5.21\,\varepsilon + 8.22\,\delta_T < 0.393 \qquad \textbf{(26)}$$

Profile measurements of AUSROC II fins indicate a mean misalignment angle of just less than 0.5 degree for a pair of opposite fins. At this stage, thrust misalignment has not been ascertained, so a value of 0.5 degree will be allowed for here. Putting $\varepsilon = \delta_T = 0.5$ degree = 0.00873 radian and substituting into inequalities (25) and (26) gives:

$$155\,v < 1573$$

and

$$0.0318\,v < 0.294$$

The maximum value of cross wind speed v satisfying these conditions is 9.0 m/s.

Summarising these results, we have found that for a thrust misalignment angle of 0.5 degree, a fin misalignment angle of 0.5 degree, and a maximum cross wind speed of 9.0 m/s, AUSROC II will remain within a 45 degrees arc down range of the launcher. For zero thrust misalignment the maximum cross wind speed allowable becomes 11.0 m/s.

3.0 Rolling Motion

It is important to check that the rolling and pitching frequencies of a rocket do not have values which remain close to each other during flight, otherwise resonance may occur. This spin-yaw resonance has the effect of enlarging rocket angle of attack and the result could be disastrous.

The pitching rate in radians per metre can be estimated from equation 1.2.13 of reference (1). Using the notation of reference (1) and section 2 of this report:

pitching rate: $\theta' = 2\pi/\sigma = v(K_m \rho d^3/ I) = v(\pi\, C_{m\alpha}\rho d^3/8\, I)$ **(27)**

This is a quasi steady approximation in that it ignores the rates of change of I, K_m and ρ in solving the moment equation. These rates of change are relatively small and the approximation is reasonable.

Similar approximations can be made to estimate the rolling rate. The approach will be briefly outlined here and the calculated results presented. First, a differential equation for roll rate is set up by equating the rate of change of angular momentum to the fin damping moment plus the forcing moment due to fin misalignment. The rate of change of angular momentum component due to the change in rolling moment of inertia, I_{xx}, is small enough to be neglected. The independent variable is changed from time t to flight path distance s, and constant linear acceleration is assumed for flight during burning.

This results in the following first order differential equation for the roll rate:

$$\phi'' + ((1/2s) + a) = b \qquad \textbf{(28)}$$

$$a = (\pi\rho d^4 \;/16\, I_{xx})\, C_{lp}$$
$$b = (\pi\rho d/16\, I_{xx})\, (dC_{nf}/d\delta)\, \mathbf{y_f}\, \Sigma\delta_i$$

where:

where:	ϕ	= roll angle ($\phi'= d\phi/ds$, $\phi''= d^2\phi/ds^{2)}$
	C_{lp}	= roll damping coefficient for the fins.
	$dC_{nf}/d\delta$	= normal force coefficient derivative for a pair of fins
	$\mathbf{y_f}$	= spanwise position of fin centre of pressure measured from the longitudinal axis of rocket. (Assumed constant at 0.23m)
	$\Sigma\delta_i$	= algebraic sum of fin misalignment angles contributing to roll.
	I_{xx}	= Moment of inertia in roll. Assumed to vary linearly with time from 2.36 kgm^2 at t=0, to 1.63 kgm^2 at t>20 s.

Two approximate solutions for ϕ' can be derived from equation (28). One for the initial motion and one for the quasi steady state. On leaving the launcher at s = p, the rocket has no rolling motion but, subsequently, the roll rate builds up to some quasi steady value approximated by the roll rate at which the fin damping moment just balances the forcing moment. The initial rolling behaviour is determined by taking a and b to be constants in equation (28). The aerodynamic coefficients are taken as constant at their subsonic values and sea level density is used. Under these conditions, the solution of equation (28) is:

$$\phi' = (b/a) - (b/a)\,F(v(as))/v(as)$$

$$- (b/a)\,[\exp\{-a(s-p)\}/v(as)][v(ap) - F\{v(ap)\}] \qquad \textbf{(29)}$$

where: $F(w) = \exp(-w^2)\int_0^w \exp(y^2)\,dy$

and is tabulated in Table 3 of reference (1).

The quasi steady solution for s large, is taken as:

$$\phi' = b/a = (\mathbf{y_f}\,/\,d^2C_{lp})\,(dC_{nf}/d\delta)\Sigma\delta_i \qquad \textbf{(30)}$$

The following table shows a comparison of roll rate and pitching rate calculated for AUSROC with the motor burning. Fin profile measurements indicate a roll torque equivalent to one fin inclined at 10 minutes to the body longitudinal axis. ie $\Sigma\delta_i$ = 10 minutes = 0.0029 radian.

Table 1.

Comparison of roll and pitch rates for AUSROC during motor burning.

time (t) seconds	Pitchrate θ' rad m/s (x100)	Rollrate φ' rad m/s (x1000)
0.675	0-	0.000
0.807	-0	0.046
0.908	0-	0.168
1.000	5.0	-
1.410	-0	0.777
3.018	-0	2.360
3.000	5.1	-
4.034	-0	2.800
5.000	5.2	-
7.000	5.3	3.300
9.000	5.9	3.500
10.00	6.4	3.600
13.00	5.2	3.000
17.00	4.0	2.700
20.00	3.2	2.600

Table 1 shows that the pitch rate is at least one order of ten greater than the roll rate during burning for AUSROC II with a fin misalignment equivalent to a fin cant of 10 minutes on one fin. Roll angle and roll rate are proportional to fin cant angle, hence a fin misalignment equivalent to a cant of 100 minutes on one fin would lead to problems with roll-yaw resonance.

After motor burn-out, AUSROC II coasts to apogee where the Mach number is 0.9 and the altitude is 13000m. During this phase of flight, there is little change in pitch rate. This is because pitch rate is proportional to the square root of C_m and the decrease in density ρ is compensated by an increase in C_m which approaches its maximum value at transonic Mach numbers. The pitch rate at apogee is 3.1×10^{-2} rad / m. On the other hand, roll rate is not dependent on density but only on Mach number and, as shown in Table 1, the maximum value is 3.6×10^{-3} rad / m. Hence it can be concluded that AUSROC will be free from roll-yaw resonance with an effective fin misalignment of 10 minutes in roll.

An estimate of roll angle during initial flight can be made by integrating equation (29). Integrals of the F functions are given in Table 3 of reference (1) and the other functions involved are error functions which are tabulated in most text books on statistics. An esimate of roll angle during initial flight over two wavelenghs in yaw is shown in table 2 below.

Table 2

Variation of roll angle over first two wavelengths in yaw.

Flight path distance wavelengths in Yaw	Roll Angle Deg
0.00	0.00
0.25	0.43
0.50	2.13
0.75	4.72
1.00	7.98
1.25	11.72
1.50	15.82
1.75	20.23
2.00	24.84

Table 3 shows that for AUSROC II with 10 minutes of effective fin cant the change in roll angle is about 25 degrees over the first two wavelengths in yaw. This amount of roll would not significantly change the bias in dispersion due to fin or thrust misalignment. The cosine of 25 degrees is about 0.9 so any reduction in bias has to be less than 10%.

4.0 Divergence

A rocket body is an elastic structure and accordingly will bend when subjected to aerodynamic loading. It is possible for distortion due to bending to cause a further increase in aerodynamic loading during flight. When the stiffness of a structure is insufficient to resist bending due to this extra aerodynamic loading, a state of instability is reached, which could result in severe distortion or fracture. This state is referred to as divergence.

All that will be attempted here is the derivation of a simple criterion to provide a rough check on the adequacy of AUSROC body stiffness to resist bending arising from aerodynamic loading. To this end, the body of the rocket will be treated as a simple elastic beam of constant stiffness EI, uniform mass distribution m, and of length s. Aerodynamic loading will be represented by a normal force at each end of the beam, one generated by the fins and the other by the nose. The coordinate system used is shown in figure 1, where beam deflection is regarded as positive in the y-direction. R_o and R_s are the aerodynamic normal forces generated by the fins and nose respectively when the rocket suddenly finds itself at an angle of attack due to a cross wind gust in the side plane. The rocket responds with angular acceleration $d^2\theta/dt^2$ and with translational acceleration d^2x/dt^2 of its centre of gravity.

Figure 1. Representation of rocket body as a simple beam.

To determine the inertial loading at a point x on the beam, rigid body dynamics are assumed. Thus if x_g is the position of the centre of gravity, the acceleration of the body at position x is:

$$-(d^2x/dt^2) - (x - x_g)(d^2\theta/dt^2)$$

in the y-direction. In order to determine the slope of the beam due to bending it is necessary to first derive an expression for the bending moment at x. To do this, a segment of beam of length x, starting from x=0, is isolated from the rest of the beam by an imaginary cut. At the cut a force R_x and a moment M_x are introduced. These represent the internal force and moment at position x, imposed by the portion of the beam to the right of the cut upon the portion to the left. This force and moment system is shown in figure 2.

The mass of the beam segment is mx and the acceleration of the centre of gravity at 0.5 x is:

$$-(d^2x_g/dt^2) - ((x/2) - x_g)(d^2\theta/dt^2) \text{ (in the y-direction)}$$

Figure 2. Force and moment system on segment of beam from 0 to x.

The resulting force equation is thus,

$$mx\,(-(d^2x_g/dt^2) + (\,(x/2) - x_g\,)\,(d^2\theta/dt^2)\,) \qquad \textbf{(30)}$$

The moment equation for the segment will now be derived.The moment of inertia of the beam segment about its centre of gravity at 0.5x is $mx^2/12$ and the angular acceleration is $d^2\theta/dt^2$. The moment about 0.5x, in the direction of θ, is:

$$(R_x - R_o)\,(x/2) + M_x$$

Thus: $$(mx^3/12)\,(d^2\theta/dt^2) = (R_x - R_o)\,(x/2) + M_x \qquad \textbf{(31)}$$

Because the beam is unrestrained at each end, $M_x=0$ at x=0 and x=s. Putting x=s and $x_g=0.5s$, in equations (30) and (31) gives:

$$m(d^2x_g/dt^2) = (1/s)\,(R_o + R_s)$$

$$m(d^2\theta/dt^2) = (6/s^2)\,(R_s - R_o)$$

Substituting these values of $m(d^2x_g/dt^2)$ and $m(d^2\theta/dt^2)$ into equations (30) and (31) and eliminating R_x, gives:

$$M_x = R_s\,(x^2/s^2)\,(s - x) + R_o\,(x/s^2)\,(s - x)^2$$

as the internal bending moment at position x along the beam. According to simple beam bending theory:

$$EI\,y'' = -M_x \qquad \text{where ' indicates } d/dx$$

This equation is integrated to find the relationship between the slope of the beam, y', at each end. Thus:

$$EI\,y'(s) - EI\,y'(o) = -\int_0^s \left(R_s\,(x^2/s^2)\,(s - x) + R_o\,(x/s^2)\,(s - x)^2\right) dx$$

$$= -\,(s^2/12)\,(R_s + R_o) \qquad \textbf{(32)}$$

This bending of the beam changes the angle of attack at each end. If the angle of attack of the beam before bending is α, then after bending:

$$\alpha_o = \alpha - y'(o) \qquad \alpha_s = \alpha - y'(s)$$

and hence: $$y'(s) - y'(o) = \alpha_o - \alpha_s \qquad \textbf{(33)}$$

where: α_o = the angle of attack of the fins at x = 0
α_s = angle of attack of the nose at x = s

Thus, the aerodynamic forces generated by these angles of attack are:

$$R_o = Q\,S\,(dC_{Nf}/d\alpha)\,\alpha_o \qquad \textbf{(34)}$$

$$R_s = Q\,S\,(dC_{Nn}/d\alpha)\,\alpha_s \qquad \textbf{(35)}$$

where: Q = dynamic pressure $0.5\,\rho\,V^2$
$dC_{Nf}/d\alpha$ = aerodynamic normal force coefficient derivative for the fins
$dC_{Nn}/d\alpha$ = aerodynamic normal force coefficient derivative for the nose
S = reference area on which the aerodynamic coefficients are based (ie $\pi d^2/4$ for the aerodynamic prediction program)

Substituting for $y'(s) - y'(o)$, R_s, and R_o, from equations (33), (34), (35), equation (32) gives the following relationship between α_s and α_o:

$$\alpha_s = \alpha_o \frac{1 + [\,Q\,S\,(dC_{Nf}/d\alpha)\,s^2\,] \,/\, 12\,EI}{1 - [\,Q\,S\,(dC_{Nn}/d\alpha)\,s^2\,] \,/\, 12\,EI} \qquad \textbf{(36)}$$

The rocket body at the fins will be extremely stiff compared with the rest of the body. Hence most of the initial bending in response to a gust will tend to increase the nose incidence. Thus if we interpret α_o as unbent body angle of attack in equation (36) and α_s as nose angle of attack after bending, we have a conservative criterion for the divergent effect of increased aerodynamic loading due to body bending. This divergent effect is shown up by the denominator in equation (36) where it can be seen that as:

$$[\,Q\,S\,(dC_{Nn}/d\alpha)\,s^2\,] \,/\, 12\,EI \rightarrow 1$$

α_s becomes very large.

Often, the design criterion for determining the structural strength of a rocket body is that it should withstand the aerodynamic loading arising from a given dynamic pressure and unbent angle of attack. Because of the effect of bending, the actual angle of attack that can be withstood will be somewhat less. Suppose that this reduced angle of attack, which must not be exceeded in flight, is α/n. Then, putting $\alpha_o = \alpha/n$ and $\alpha_s < \alpha$, in equation (36), gives the criterion to be satisfied by the stiffness EI as:

$$EI > [(Q S s^2) / 12(n-1)] [(dC_{Nf}/d\alpha) + n(dC_{Nn}/d\alpha)] \qquad \textbf{(37)}$$

For AUSROC II, maximum dynamic pressure is 186 kPa occurring at all burnt when M=2.37. Thus:

$Q = 186$ kPa
$dC_{Nn}/d\alpha = 3.6$
$dC_{Nf}/d\alpha = 9.85$
$S = \pi d^2/4 = 0.0491$ m^2
s = body distance between centres of pressure of fin and nose normal forces. (Taken as 5m)

With these values for AUSROC, criterion (37) becomes:

$$EI > [19.02 / (n-1)] (9.85 + 3.6n) \text{ kPa m}^4 \qquad \textbf{(38)}$$

Putting n=2 for example in criterion (38) tells us that for AUSROC II if:

$$EI > 324.3 \text{ kPa m}^4$$

the nose angle will not exceed twice the unbent angle of attack.

5. Normal Force Distribution on Fins

In this section approximate expressions for the normal force distribution on a fin are given for the condition of maximum dynamic pressure. This occurs at all burnt when the Mach number is 2.37 according to the particle trajectory computation for the drag factor of 1. By taking the value of Mach number at all burnt to be 2.539, which is close to the computed value for a drag correction factor of 0.8, the Mach line geometry on the fin surface is simplified. At M=2.539 the Mach lines emanating from the root and tip leading edge intersect at the fin trailing edge. This divides the fin surface up into three regions as shown in figure 3.

Figure 3. Fin planform showing three regions of supersonic flow.

The procedure adopted here to derive approximate expressions for fin normal force distribution is that at first the well-known results of linearised supersonic flow theory will be used to predict lifting pressure coefficients on the boundaries of regions 1, 2 and 3. These pressure coefficients take no account of the presence of the body. The body effect is taken into account by multiplying the fin alone pressure coefficients by the Beskin upwash factor $(1 + r^2 / (r + y)^2)$. The pressure coefficients between the boundaries are then assumed to vary linearly in the spanwise y-direction.

Region 1: In this region the air flow is conical with the property that flow conditions are constant along rays emanating from the fin leading edge junction with the root chord at O in figure 3. For the pressure coefficient along the root chord, conical supersonic flow theory gives:

$$C_p = [4\alpha / \sqrt{(\beta^2 - 1)}]\,(2/\pi)\,\sin^{-1}[\sqrt{(1 - 1/\beta^2)}]$$

where: $\beta^2 = M^2 - 1$
α = fin angle of attack in radians.

For: $M = 2.539$
$\beta = 2.33$
$Cp = 1.36$ (along the root chord y=0, for the fin alone)

When y=0, the body upwash factor: $(1 + r^2 / (r + y)^2) = 2$

Hence along the root chord y = 0: Cp = 2 x 1.36 = 2.72

Region 2: The pressure coefficient is that for a swept wing of infinite span and, for the fin alone, is given by:

$$Cp = 4\alpha / v(\beta^2\text{-}1) = 1.9\ \alpha$$

The inboard boundary of region 2 is the Mach line OR shown in figure 3 and has the equation y = 3x / 7. Substituting this value for y into the upwash factor gives:

$$Cp = 1.9\ (1 + 1/(1 + 0.024x / 7)^2)\alpha$$

as the pressure coeficient along the Mach line OR.

The other two boundaries of region 2 are the leading edge y = x and the Mach line PR with equation:

$$y = -3x / 7 + 3000/7$$

Thus: $$Cp = 1.9\ (1 + 1/(1 + 0.008x)^2)\alpha$$

is the pressure coefficient along the leading edge y = x, and:

$$Cp = 1.9\ (1 + 1/(1 - 0.024x / 7 + 24/7)^2)\alpha$$

is the pressure coefficient along the Mach line PR.

Region 3: This is a region of tip flow and along the fin tip PQ in figure 3 Cp=0. Thus to find the value of Cp at some point (X,Y) on the fin we need to establish in which region the point lies. Then establish which two boundaries the point is between in the y-ward directions and linearly interpolate between the values of Cp on each boundary. For example, if

$$Y > 3X / 7 \text{ and } X < 300$$

then the point lies in region 2 between y = 3X / 7 and y = X on the spanwise line x = X. Then by linear interpolation:

$$Cp(X,Y) = Cp(X,3X/7) + [(Y\text{-}3X/7)/(X\text{-}3X/7)]\ (Cp(X,X) - Cp(X,3X/7))$$

where: $Cp(X,3X/7) = 1.9(1 + 1/(1 + 0.024X/7)^2)\alpha$ on the Mach line OR

and: $Cp(X,X) = 1.9(1 + 1/(1 + 0.008X)^2)\alpha$ on the fin leading edge.

As a check on the error involved in this way of approximating Cp(X,Y), an integration taken over the entire fin gave a value of $S(dC_n/d\alpha)$ 20% greater than the value obtained from the aerodynamic prediction program. To find the aerodynamic normal force over an area of fin surface, integrate Cp(x,y) over the area, using the approximate relationships, and multiply the result by the free stream dynamic pressure, which at all burnt is 186kPa. The foregoing results simplify calculations of fin loadings and moments needed for checking fin strength and are conservative because they overestimate fin loading.

6. Results and Conclusions

This section provides a summary of the results of the investigations described in this Report, starting with rocket flight path dispersion.

6.1 Lateral Dispersion

Calculations have been made to estimate lateral flight path dispersion due to cross wind, fin misalignment and thrust misalignment. Lateral displacements from the unperturbed trajectory at all burnt have been calculated for these three causes. At all burnt, on the unperturbed trajectory, AUSROC II is at an altitude of 6000m and at a down range horizontal distance of 5000m from the launcher. While the motor is thrusting, a steady cross wind causes a rocket to deviate towards the direction from which the cross wind is coming. Thus for a cross wind blowing from the right, looking down range, a thrusting rocket will veer towards the right. The calculated results for lateral displacement from the unperturbed trajectory at all burnt are as follows:

149m for every m/s of cross wind

389m for every degree of fin misalignment

792m for every degree of thrust misalignment

The last two figures have been calculated on the assumption that the rocket does not roll and hence that the plane in which the motor thrust is offset, due to the misalignment, remains fixed.

The results for the lateral angular displacement of the perturbed trajectory from the unperturbed trajectory at motor all burnt are:

1.22 degrees for every m/s of cross wind

3.62 degrees for every degree of fin misalignment

5.71 degrees for every degree of thrust misalignment

These lateral angular deviations for fin and thrust misalignments occur in the plane in which the thrust is offset for a non-rolling rocket. For the cross wind, the angular deviation is in the side plane, defined as the plane which contains the tangent to the unperturbed trajectory and which is normal to the vertical plane. Hence the side plane orientation changes as the unperturbed trajectory tangent changes direction along the flight path. At motor all burnt, the unperturbed trajectory tangent makes an angle of 46 degrees with the horizontal. Because range boundaries are defined in the horizontal ground plane, the angular deviations quoted for the side plane need to be projected on to the horizontal plane.

The relationship between these angles is that:

Tan(Side plane angle) = Tan(Projection of angle in horizontal plane) x Cos(Angle between trajectory tangent and horizontal).

Thus for small deviation angles we can drop the Tan from the above relationship. Then at all burnt:

(Side plane angle) = (Projection of angle in horizontal plane)
x Cos(46degrees)

Lateral angular deviation is a maximum when the deviations due to fin and thrust misalignment occur in the same direction as that due to cross wind, which is in the side plane defined above. In this case, the projections of the deviations on to the horizontal plane at motor burn out are:

1.76 degrees for every m/s of cross wind

5.2 degrees for every degree of fin misalignment

8.22 degrees for every degree of thrust misalignment

These results can be used to place a conservative restriction on cross wind speed. Taking the measured fin misalignment angle of 0.5 degree and allowing 0.5 degree for thrust misalignment it is found that for cross wind speeds of less than 9 m/s, AUSROC II will remain within a 45 degree arc down range of the launcher.

6.2 Dispersion due to Tail Wind

The significance of a tail wind is that it tends to increase the effective launch angle of a thrusting rocket. For launch angles close to 90 degrees, lateral dispersion entirely governs the initial direction of flight, so that flight in a backwards direction behind the launcher is a possibility. The calculated result for tail wind dispersion is that the trajectory elevation angle is increased by 1.15 degrees for every m/s of tail wind. A conservative limit of 10 m/s is suggested for tail wind. This would increase the QE from 70 degrees to an effective 81.5 degrees, leaving some margin for fin and thrust misalignment.

Dispersion is sensitive to the value of rocket speed as the rocket just leaves the launcher. It is important to ensure that AUSROC II develops its full design speed of 30 m/s before leaving the launcher. A loss of speed due to slow thrust build up could greatly increase dispersion.

6.3 Rolling Motion

Rolling rate is proportional to the effective fin cant which from fin measurements was found to be equivalent to a cant of 10 minutes on one fin panel. This produces a roll rate of about 0.1 of the pitch rate. Hence it is concluded that roll-yaw resonance will not be a problem. An effective fin cant of about 100 minutes on one fin panel would lead to problems with roll-yaw resonance.

Most of the dispersion due to wind,fin and thrust misalignment, occurs over the first two wavelengths in yaw after a rocket leaves the launcher. Over this distance, the roll angle turned through by AUSROC II is calculated to be 25 degrees for the measured fin cant of 10 minutes. This is insufficient to significantly overcome the initial bias in the direction of dispersion due to thrust or fin misalignments. Roll measurements taken during the flight test of AUSROC II would be necessary for analysing results from onboard accelerometer measurements and for checking the actual amount of fin cant.

6.4 Body Divergence

The criterion derived for body stiffness EI, to avoid divergence due to elastic deformation under aerodynamic loading is that:

$$EI >> (Q\ S\ s^2\ /12)\ (dC_{Nn}/d\alpha)$$

where: $dC_{Nn}/d\alpha$ = normal force coefficient derivative for nose
S = reference area for aerodynamic coefficient = $\pi d^2/4$ =.0491 m^2
s = body length = 5m
Q = dynamic pressure

For the maximum dynamic pressure which occurs at all burnt:

$Q = 186$ kPa $\qquad dC_{Nn}/d\alpha = 3.6$

and the above criterion becomes:

$$EI >> 68.5 \text{ kPa m}^4$$

The closer EI approaches $(Q\ S\ s^2\ /12)\ (dC_{Nn}/d\alpha)$ the greater the tendency of the body to bend without limit and hence for the body to fracture under aerodynamic loading.

Another derived result which relates the amount of bending to the stiffness EI is as follows. When a wind gust induces a given angle of attack at the fins, which form a relatively stiff part of the body structure, the angle of attack of the nose after body bending will be less than n times the fin angle of attack if:

$$EI > (Q\ S\ s^2\ /\ (n-1)\ )\ (dC_{Nf}/d\alpha\ +\ ndC_{Nn}/d\alpha)$$

which for n = 2 at maximum dynamic pressure becomes:

$$EI > 324 \text{ kPa m}^4$$

6.5 Aerodynamic Load Distribution on Fins

Simplified expressions for fin normal force distribution are given in section 5. These expressions are derived from linear interpolations between values calculated from linear supersonic flow theory with the upwash effect from the body superimposed. Being analytically simple, these expressions are suggested for use in fin stress calculations where bending moments need to be determined.