**PCD04D4 Features**

**Turbo Decoder**
- 16 state DVB–RCS2 compatible
- Rate 1/2 or 1/3
- 40 to 4800 bit interleaver
- Up to 192 MHz internal clock
- Up to 136 Mbit/s with 5 decoder iterations
- 6–bit signed magnitude input data
- 4 parallel MAP decoders
- Optional log–MAP or max–log–MAP constituent decoder algorithms
- Up to 32 iterations in 1/2 iteration steps
- Optional power efficient early stopping
- Optional extrinsic information scaling and limiting
- Estimated channel error output
- Free simulation software

- Available as VHDL core for ASICs

**Introduction**

The PCD04D4 is a compatible DVB–RCS2 [1] error control decoder. DVB–RCS2 uses a 16 state rate 2/4 duo–binary tail–biting turbo code with an almost regular permutation (ARP) interleaver.

For DVB–RCS2, there are 24 interleaver sizes ranging from 112 to 4792 bits. Five parameters $P$, $Q_0$, $Q_1$, $Q_2$, and $Q_3$ are used by the interleaver. The decoder uses a simplified version of the interleaver with four parameters.

For DVB–RCS2 a code rate of 1/3 is specified. This code is punctured to obtain lower rates. The code uses a 16 state rate 2/4 systematic recursive convolutional tail–biting constituent code. Since a tail–biting code is used, there are no tail bits, increasing the bandwidth efficiency of the code.

Four MAP04D MAP decoder cores are used with the PCD04D4 core to iteratively decode the DVB–RCS2 turbo code. The log–MAP algorithm for maximum performance or the max–log–MAP algorithm for minimum complexity and highest speed can be selected. The extrinsic information can be optionally scaled and limited with each half iteration, improving performance with max–log–MAP decoding.

The reverse sliding block algorithm is used with sliding block lengths of $L = 32$ or 64. To reduce MAP decoder delay by approximately one half, the data is input and output in reversed blocks of $L$ for $K/8 > L$, where $K$ is the data length in bits. Six–bit quantisation is used for maximum performance.

The turbo decoder can achieve up to 136 Mbit/s with 5 iterations and max–log–MAP decoding using a 192 MHz internal clock ($K = 4792$). Log–MAP decoding decreases speed by about 30%. Optional early stopping allows the decoder to greatly reduce power consumption with little degradation in performance.
Figure 1 shows the schematic symbol for the PCD04D4 decoder. The EDIF core can be used with Xilinx Integrated Software Environment (ISE) or Vivado software to implement the core in Xilinx FPGA’s. A VHDL simulation core is also provided. For other FPGA families and VHDL ASIC licenses, a VHDL core is provided.

Table 1 shows the resources used for various Virtex–4 and Virtex–5 devices. Resources for Virtex–II and Spartan–3 devices are similar to that for Virtex–4. Resources for Virtex–6, Spartan–6 and 7–Series devices are similar to that for Virtex–5. The MODE[1:0] inputs can be used to select various decoder implementations. The input/output memory is not included. Only one global clock is used. No other resources are used.

Table 2 shows the performance achieved with various Xilinx parts. T_{cp} is the minimum clock period over recommended operating conditions. These performance figures may change due to device utilisation and configuration.

**Table 1: Resources used.**

<table>
<thead>
<tr>
<th>Configuration</th>
<th>Virtex–4 LUTs</th>
<th>Virtex–5 LUTs</th>
<th>Block RAMS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Max–log–MAP</td>
<td>42,068</td>
<td>34,226</td>
<td>16</td>
</tr>
<tr>
<td>Log–MAP</td>
<td>58,293</td>
<td>49,277</td>
<td>16</td>
</tr>
</tbody>
</table>

**Table 2: Performance of Xilinx parts.**

<table>
<thead>
<tr>
<th>Xilinx Part</th>
<th>T_{cp} (ns)</th>
<th>f (MHz)</th>
<th>f_{d} (Mbit/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>XC4VLX60–10</td>
<td>10.954</td>
<td>91.2</td>
<td>64.8</td>
</tr>
<tr>
<td>XC4VLX60–11</td>
<td>9.343</td>
<td>107.0</td>
<td>75.9</td>
</tr>
<tr>
<td>XC4VLX60–12</td>
<td>8.299</td>
<td>120.4</td>
<td>85.5</td>
</tr>
<tr>
<td>XC5VLX85–1</td>
<td>10.274</td>
<td>97.3</td>
<td>69.1</td>
</tr>
<tr>
<td>XC5VLX85–2</td>
<td>8.790</td>
<td>113.8</td>
<td>80.8</td>
</tr>
<tr>
<td>XC5VLX85–3</td>
<td>7.853</td>
<td>127.3</td>
<td>90.4</td>
</tr>
<tr>
<td>XC6VLX75T–1</td>
<td>8.775</td>
<td>114.0</td>
<td>80.9</td>
</tr>
<tr>
<td>XC6VLX75T–2</td>
<td>7.564</td>
<td>132.2</td>
<td>93.8</td>
</tr>
<tr>
<td>XC6VLX75T–3</td>
<td>6.796</td>
<td>147.1</td>
<td>104.4</td>
</tr>
<tr>
<td>XC7A100T–1</td>
<td>10.828</td>
<td>92.4</td>
<td>65.6</td>
</tr>
<tr>
<td>XC7A100T–2</td>
<td>8.876</td>
<td>112.7</td>
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<tr>
<td>XC7K160T–1</td>
<td>6.925</td>
<td>144.4</td>
<td>102.5</td>
</tr>
<tr>
<td>XC7K160T–2</td>
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<td>175.9</td>
<td>124.9</td>
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Max–Log–MAP, 5 iterations, K = 4792, L = 32

12^∗ = length 1552 (194 bytes)  
13^∗ = length 984 (123 bytes)  
14 = length 1504 (188 bytes)  
15 = length 2112 (264 bytes)  
16^∗ = length 2384 (298 bytes)  
17 = length 2684 (333 bytes)  
18^∗ = length 2840 (355 bytes)  
19 = length 3200 (400 bytes)  
20 = length 3552 (444 bytes)  
21^∗ = length 4312 (539 bytes)  
22 = length 4792 (599 bytes)  
32,33 = length 800 (100 bytes)  
34–36^∗ = length 1360 (170 bytes)  
37–39^∗ = length 3504 (438 bytes)  

* While DEC_END = 0, XD valid for even NA only.

**Signal Descriptions**

C  MAP Decoder Constant (0–11)  
CLK  System Clock  
DEC_END  Decode End Signal  
ERR  Estimated Error  
P0I–P3I  Interleaver parameters (used when KS = 0).  
P0I[6:1] = P div 2  
P1I[11:2] = Q1  
P2I[11:2] = (Q0P + Q2) mod k/8  
P3I[11:2] = (Q0P + Q3) mod k/8  
K  Data Length (K = 40–4800)  
K[12:3] = K/8  
KS  KS Data Length Select  
0 = select K, P0I–P3I  
1.3^∗ = length 304 (38 bytes)  
2^∗ = length 112 (14 bytes)  
4^∗ = length 472 (59 bytes)  
5^∗ = length 680 (85 bytes)  
6 = length 768 (96 bytes)  
7 = length 864 (108 bytes)  
8^∗ = length 920 (115 bytes)  
9^∗ = length 1040 (130 bytes)  
10 = length 1152 (144 bytes)  
11^∗ = length 1400 (175 bytes)  

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* While DEC_END = 0, XD valid for even NA only.
Table 3: MODE selection

<table>
<thead>
<tr>
<th>Input</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>MODE0</td>
<td>0 = max–log–MAP</td>
</tr>
<tr>
<td></td>
<td>1 = log–MAP</td>
</tr>
<tr>
<td>MODE1</td>
<td>0 = rate 1/2</td>
</tr>
<tr>
<td></td>
<td>1 = rate 1/3</td>
</tr>
</tbody>
</table>

Table 3 describes each of the MODE[1:0] inputs that are used to select various decoder implementations. Note that MODE[1:0] are “soft” inputs and should not be connected to input pins or logic. These inputs are designed to minimise decoder complexity for the configuration selected.

Note that the required size of each of the 16 internal interleaver memories is 150x28. This is implemented using 16 512x36 Xilinx BlockRAMs. Although the nominal maximum data length is 4800 bits, the decoder can actually decode up to 8128 bits for SLD = 0 or 8152 bits for SLD = 1.

Turbo Decoder Parameters

For optimal performance, the maximum a posteriori (MAP) [2] constituent decoder can be used which is dependent on the signal to noise ratio (SNR). Unlike other turbo decoders with sub-optimum soft-in–soft-in (SISO) decoders, using the MAP (or specifically the log–MAP [3]) algorithm can provide up to 0.5 dB coding gain at low SNRs. Log–MAP operation is implemented when MODE0 is high.

With binary phase shift keying (BPSK, \(m = 1\)) or quadrature phase shift keying (QPSK, \(m = 2\)) modulation (see Figure 2) the decoder constant \(C\) should be adjusted such that

\[
C = A \sigma^2 / \sqrt{m} / 2. \tag{1}
\]

where \(A\) is the signal amplitude and \(\sigma^2\) is the normalised noise variance given by

\[
\sigma^2 = 1 / ((2mR_E N_0)). \tag{2}
\]

\(E_b/N_0\) is the energy per bit to single sided noise density ratio, \(R = 1/n\) is the code rate, \(K\) is the data length, and \(n = 2\) or 3. \(C\) should be rounded down to the nearest integer and limited to be no higher than 11. Max–log–MAP [3] operation occurs when \(C = 0\). Due to quantisation effects, \(C = 1\) is equivalent to \(C = 0\). Thus \(C = 1\) is internally rounded down to \(C = 0\). Also, as \(C = 2\) or 3 give poor performance due to quantisation effects, these values are internally rounded up to \(C = 4\). Max–Log–MAP operation is implemented when MODE0 is low.

For each code (with a particular block size, rate and number of iterations), there will be a minimum \(E_b/N_0\) where the maximum acceptable BER or FER is achieved. The value of \(C\) should be chosen for this \(E_b/N_0\). This value of \(C\) can be kept constant for all \(E_b/N_0\) values for this code. For higher values of \(E_b/N_0\), there will be negligible degradation in performance, even though \(C\) will be higher than optimal [4]. For lower \(E_b/N_0\) values, there could be up to a few tenths of a dB degradation, since \(C\) will be lower than optimal. However, this should not have much impact since the BER or FER will already be above the maximum acceptable level anyway.

For fading channels the value of \(A\) and \(\sigma^2\) should be averaged across the block to determine the average value of \(C\). Each received value \(r_k\) should then be scaled by \((A r^2) / (A_k \sigma_k^2)\) where \(A_k\) and \(\sigma_k^2\) are the amplitude and normalised variance of \(r_k\). Note that this scaling should be performed for both the log–MAP and max–log–MAP algorithms for optimal performance.

The value of \(A\) directly corresponds to the 6–bit signed magnitude inputs (shown in Table 4). The
6–bit inputs have 63 quantisation regions with a central dead zone. The quantisation regions are labelled from –31 to +31. For example, one could have \( A = 15.7 \). This value of \( A \) lies in quantisation region 16 (which has a range between 15.5 and 16.5).

**Table 4: Quantisation for R0I[5:0], etc.**

<table>
<thead>
<tr>
<th>Decimal</th>
<th>Binary</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>31</td>
<td>011111</td>
<td>30.5−∞</td>
</tr>
<tr>
<td>30</td>
<td>011110</td>
<td>29.5−30.5</td>
</tr>
<tr>
<td>2</td>
<td>000010</td>
<td>1.5−2.5</td>
</tr>
<tr>
<td>1</td>
<td>000001</td>
<td>0.5−1.5</td>
</tr>
<tr>
<td>0</td>
<td>000000</td>
<td>−0.5−0.5</td>
</tr>
<tr>
<td>33</td>
<td>100001</td>
<td>−1.5−0.5</td>
</tr>
<tr>
<td>34</td>
<td>100010</td>
<td>−2.5−1.5</td>
</tr>
<tr>
<td>62</td>
<td>111110</td>
<td>−30.5−29.5</td>
</tr>
<tr>
<td>63</td>
<td>111111</td>
<td>−∞−30.5</td>
</tr>
</tbody>
</table>

Since most analogue to digital (A/D) converters do not have a central dead zone, a 7–bit A/D should be used and then converted to 6–bit as shown in the table. This allows maximum performance to be achieved.

For signed magnitude inputs a decimal value of 32 has a magnitude of 0 (equivalent to −0). External two’s complement values will need to be converted to sign magnitude for input to the decoder. Note that for two’s complement decimal 32 (integer −32), this needs to be limited to decimal 33 (integer −31).

For input data quantised to less than 6–bits, the data should be mapped into the most significant bit positions of the input, the next bit equal to 1 and the remaining least significant bits tied low. For example, for 3–bit received data \( R0T[2:0] \), where \( R0T[2] \) is the sign bit, we have \( R0I[5:3] = R0T[2:0] \) and \( R0I[2:0] = 4 \) in decimal (100 in binary). For punctured input data, all bits must be zero, e.g., \( R1I[5:0] = 0 \).

Due to quantisation and limiting effects the value of \( A \) should also be adjusted according to the received signal to noise ratio.

**Example 1:** Rate 1/3 BPSK code operating at \( E_b/N_0 = 0.3 \) dB. From (2) we have \( \sigma^2 = 1.39988 \). Assuming \( A = 9.0 \) we have from (1) that \( C = 6 \) rounded down to the nearest integer.

Figure 3 gives a simplified block diagram of the PCD04D4 16 state turbo decoder. The extrinsic information output for each of the four data symbols is given by four 9–bit values within the MAP decoder. To reduce the number of bits, two bits of ZO indicates the symbol with the smallest extrinsic information, with three eight bit values giving the scaled and limited extrinsic information of the other three symbols, minus the extrinsic information of the symbol with the smallest extrinsic.
information. This gives increased performance compared to using the extrinsic information for each of the two decoded bits, which entails a loss of information. The remaining two bits of ZO are the hard decision values of the received data.

For even half iterations (odd NA), data is decoded for the interleaved received data. This means that data must be read in interleaved order. Due to the nature of the ARP interleaver, this would imply that the input memory (which is external to the decoder) would need to be implemented using 16 separate memories, the same as for the interleaver memory. To avoid this, we add a delayed version of the received data to the decoder extrinsic information for odd half iterations. For even half iterations, only the extrinsic information is output. This implies the input memory can be implemented using a single memory.

The above also explains the two hard decision bits in ZO. These bits are needed to determine the estimated error output ERR for even half iterations.

The sliding window algorithm used depends on $K/8$ and $L$. For $K/8 = L$, Figure 4 shows how the forward and reverse state metrics (SM) are calculated. The horizontal axis shows decoder time. The vertical axis the received symbol index. An arrow going up shows forward SMs for $L$ symbols. An arrow going down shows reverse SMs for $L$ symbols. Horizontal dashed arrows going backwards indicate SMs being passed between iterations. Forward SMs are indicated by an F and reverse SMs by R. Forward SMs that have been reversed in time are indicated by E. Decoded data is output when R and E are used together.

We see that in this case the RAMs are read in forward blocks of $L$. Decoded data is written in reversed blocks of $L$. Also, the SMs are allowed to settle for $L$ symbols before being used. As $L$ can be very small, e.g., $L = 14$ for $K = 112$, this is not sufficient to obtain reliable SMs. Thus, we pass both forward and reverse SMs between iterations to help improve the reliability of the SMs.

Figures 5 to 7 show the timing diagram for various cases where $K/8 > L$. Only the trellis diagrams...
for the first two MAP decoders are shown. Unlike the previous case, we read the SMs in reverse blocks of \( L \) and output them in reverse order. We pass forward SMs between iterations so that the starting forward SMs become reliable. This is indicated by the dashed arrow going backwards in time. As the reverse state metrics are reliable when used to calculate the extrinsic information there is no need to pass reverse SMs between iterations.

However, for \( K/8 > 2L \) the final SMs of the third block, for example R3 in Figure 6, are stored and then passed to the previous MAP decoder. This is indicated by the dashed arrow going forwards in time. For \( L < K/8 < 2L \) (Figure 5), the reverse SMs for the second reverse SM calculator are passed directly between MAP decoders.

For \( L < K/8 \leq 3L \) (Figures 5 and 6), the first reverse SM calculator initially uses input data from the next MAP decoder for the last input block in time. This is why the RxI inputs shown in Figure 3 have an input width twice that of a single sample of six bits. This method avoids additional multiplexers to select the first reverse SMs from the next MAP decoder. However, this technique can not be used for \( K/8 > 3L \) as the old extrinsic information will have been overwritten with new data. Instead, we use the previously stored reverse SMs as shown by the first forward arrow in Figure 7.

For \( K/8 > 4L \) the timing diagrams are similar to that for \( 3L < K/8 \leq 4L \) (Figure 7). In all cases, we see that decoding time is equal to \( K/8 + 2L \). We also need to add an additional 12 clock cycles for pipeline delay, to give a decoding time of \( K/8 + 2L + 12 \) clock cycles for each half iteration. Two of the 12 clock cycles are for calculating the a priori information, eight clock cycles for the MAP decoder delay and two clock cycles for calculating the modified extrinsic information. One additional clock cycle is used to start the turbo decoder.

The number of turbo decoder half–iterations is given by \( NI \), ranging from 0 to 63. \( NI = 2I - 1 \) where \( I \) is the number of iterations. This is equivalent to 0.5 to 32 iterations. The decoder initially starts at half iteration \( NA = 0 \), increasing by one until \( NI \) is reached or at an earlier time if early stopping is enabled. The NA output can be used to select LIMZ and SCLZ values, which is useful for max–log–MAP decoding.

The turbo decoder speed \( f_d \) is given by

\[
f_d = \frac{F_d K}{(NI + 1)(K/8 + 2L + 12) + 1}
\]

where \( F_d \) is the CLK frequency and \( L \) is the MAP decoder sliding window length. Table 5 gives the value of \( L \) depending on \( K \) and SLD. \( SLD = 0 \) can be used to increase decoder speed, while \( SLD = 1 \) should be used for high puncturing rates to increase performance.

**Table 5: Sliding window length**

<table>
<thead>
<tr>
<th>( K )</th>
<th>( L )</th>
</tr>
</thead>
<tbody>
<tr>
<td>min</td>
<td>max</td>
</tr>
<tr>
<td>40</td>
<td>256</td>
</tr>
<tr>
<td>264</td>
<td>512</td>
</tr>
<tr>
<td>520</td>
<td>4800</td>
</tr>
</tbody>
</table>

For example, if \( F_d = 100 \) MHz and \( I = 5 \) \( (NI = 9) \) the decoder speed ranges from 20.7 Mbit/s for \( K = 112 \) and \( L = 14 \) to 71.0 Mbit/s for \( K = 4792 \) and \( L = 32 \).

An important parameter is LIMZ, the limit value for the extrinsic information. Extrinsic information is the “correction” term that the MAP decoder determines from the received data and a priori information. It is used used as a priori information for the next MAP decoding or half iteration. By limiting the correction term, we can prevent the decoder from making decisions too early, which improves decoder performance. The limit factor LIMZ should vary between 1 and 193. We recommend that 193 be used.

Another parameter that can used to adjust decoder performance is SCLZ which ranges from 1 to 32. The extrinsic information is scaled by SCLZ/32 followed by rounding to the nearest integer. Thus, when SCLZ = 32, no scaling is performed. For log–MAP decoding we recommend SCLZ = 29. For max–log–MAP decoding we recommend SCLZ = 23. The NA output can be used to adjust LIMZ and SCLZ with the number of iterations for optimum performance.

There are four decoder operation modes given by \( M \). Mode \( M = 0 \) decodes a received block with a fixed number of iterations (given by \( NI \)). Modes 1 to 3 are various early stopping algorithms. Early stopping is used to stop the decoder from iterating further once it has estimated there are zero errors in the block. Mode 1 will stop decoding after an odd number of half–iterations. Mode 2 will stop decoding after an even number of half iterations. Mode 3 will stop after either an odd or even number of half iterations. Further details are given in the next section.
Interleaver parameters
The interleaving equation is given by
\[ \pi(j) = (Pj + Q(j \mod 4) + 3) \mod K/2 \]  \hfill (4)
where \( j \) varies from 0 to \( K/2-1 \). Table 6 gives the formulas for \( Q(j) \).

Table 6: Interleaver Parameters

<table>
<thead>
<tr>
<th>( j )</th>
<th>( Q(j) )</th>
</tr>
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<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>4Q1</td>
</tr>
<tr>
<td>2</td>
<td>4Q0P + 4Q2</td>
</tr>
<tr>
<td>3</td>
<td>4Q0P + 4Q3</td>
</tr>
</tbody>
</table>

The parameters \( P \) and \( Q_0 \) to \( Q_3 \) depend on the block length \( K \). These values are given in the standard. \( P \) is an odd number while \( Q_0 \) to \( Q_3 \) can be odd or even numbers. To reduce interleaver complexity, we let \( P(j) = Q(j) \mod K/2 \) for \( j = 1 \) to \( 3 \). We have that
\[ Q(j) = D(j)K/2 + P(j) \]  \hfill (5)
where \( D(j) = Q(j) \mod K/2 \). As four divides \( Q(j) \) and \( K/2 \), four must also divide \( P(j) \). That is
\[ Q(j)/4 = D(j)K/8 + P(j)/4. \]  \hfill (6)

Since \( P(j)/4 < K/8 \) the decoder uses \( P_j = Q(j)/4 \mod K/8 \) for \( j = 1 \) to \( 3 \) for the internal parameters. The term \( Q(0) \) does not need to be externally input since it is always zero. We also let \( P_0 = P \div 2 \).

When \( KS[5:0] = 0 \), the byte length \( K/8 \) is input to \( K[12:3] \) and the interleaver parameters \( P_0 \) to \( P_3 \) are input to \( P0[6:1] \) and \( P1[11:2] \) to \( P3[11:2] \), respectively. Internally, the two least significant bits \( P[j][1:0] = 0 \), \( 1 \leq j \leq 3 \), and least significant bit \( P0[0] = 1 \).

When \( KS[5:0] > 0 \), the internal data length selected by \( KS \) (equal to the Waveform ID) is used. Also, the internal interleaver parameters \( P_0 \) to \( P_3 \) for the data length from the standard are used. The inputs \( K[12:3] \), \( P0[6:1] \), and \( P1[11:2] \) to \( P3[11:2] \) are ignored.

With four parallel MAP decoders, the interleaver addresses are given by
\[ r(i, m) = \pi(i + mK/8) \mod K/8 \]  \hfill (7)
\[ s(i, m) = \pi(i + mK/8) \div K/8 \]  \hfill (8)
where \( r(i, m) \) is the address of the depth \( K/8 \) RAM, \( s(i, m) \) selects one of the four RAMs, \( i \) is the input RAM address from 0 to \( K/8-1 \), \( m \) is the input select address from 0 to 3 and \( j = i + mK/8 \) is the input address for the interleaver. We have that
\[ r(i, m) = (Pi + Q(i + mK/8) \mod 4) + 3 \mod K/8 \]

If one of the following conditions
\[ Q(j) \mod K/8 = 0, \ 1 \leq j \leq 3 \]  \hfill (9)
\[ Q(1) \mod K/8 = Q(3) \mod K/8, \]  \hfill (10)
\[ Q(2) \mod K/8 = 0, \ K/8 \mod 2 = 0, \]  \hfill (11)
\[ K/8 \mod 4 = 0 \]  \hfill (12)
is satisfied, then \( r(i, m) = \pi(i) \mod K/8 \). This implies that all four pairs of decoded bits will have the same write address. Thus, the decoded output XD[7:0] will be valid for all values of NA[5:0]. However, if all of the above conditions are not satisfied, then XD[7:0] will only be valid for even NA[5:0] (odd half iterations) where non-interleaved decoding is performed.

Internal to the decoder, the property that \( Q(j) \) is a multiple of four is used to split the interleaver memory into 16 separate memories. This allows correct read and write operations of the extrinsic information, regardless if the above conditions are satisfied.

Turbo Decoder Operation

After the START signal is sent, the decoder will read the received data at the CLK speed. It is assumed that the received data is stored in one synchronous read RAM of size \( (K/8) \times 24n \), with \( n = 4 \) or \( 6 \) for rate \( 2/3 \) decoding.

For input \( R[6j+5:6j][k], i = 0 \) to 5, \( j = 0 \) to 3, and \( k = 0 \) to \( K/8-1 \), the input data corresponds to code symbol \( 6(K/8)j + 6k + i \). The read address for input \( R[i] \) is given by \( RA \). The received data for \( A, B, Y_1, Y_2, W_1, \) and \( W_2 \) are input to \( R01 \) to \( R51 \), respectively.

The received data ready signal RR goes high to indicate the data to be read from the address given by \( RA[9:0] \). The parity check equations for the code are
\[ (1 + D + D^2 + D^4)A + (1 + D^2 + D^4)B + (1 + D^3 + D^4)Y = 0 \]  \hfill (13)
\[ (1 + D^2 + D^3 + D^4)A + (1 + D + D^2 + D^4)B + (1 + D^3 + D^4)W = 0 \]  \hfill (14)

The decoder then iteratively decodes the received data for \( NI+1 \) half iterations, rereading the received data for each half iteration for either \( T_1 = 2L \) CLK cycles for \( K/8 = L \) or \( T_2 = L((K/8-1) \div L) + L \) CLK cycles for \( K/8 > L \). For \( K/8 \leq 3L \), the signal RR goes high for either \( T_1 \) or \( T_2 \) CLK cycles while data is being output. For \( K/8 > 3L \), RR is high for \( T_{2-L} \) CLK cycles and then goes high again at the \( (K/8) \)th CLK cycle for \( T_{2-K/8} \) CLK cycles.
Figure 8: Turbo Decoder Input Timing ($K = 112$, $L = 14$, first half iteration).

Figure 8 illustrates the decoder timing where the data is input during the first half iteration. The input $R_k$, for $k = 0$ to $K/8 - 1$ corresponds to input data $R|23:0|3[k]$, $i = 0$ to 5.

Note that while DEC_END is low (decoding is being performed), the START signal is ignored, except for the last clock cycle before DEC_END goes high. If the code is being changed, then START must wait until DEC_END goes high, otherwise, it can go high in the last clock cycle.

A synchronous reset is also provided. All flip flops in the turbo decoder are reset during a low to high transition of CLK while RST is high.

The decoded data is output during the last half iteration on XD[7:0]. That is, decoded data is output 8-bits every CLK cycle. For $k = 0$ to $K/8 - 1$ and $j = 0$ to 3 we have $XD[2j] = A_{k/8 + k}$ and $XD[2j + 1] = B_{k/8 + k}$. The signal XDR goes high for $K/8$ CLK cycles while the block is output. If $NI$ is even (odd half iterations), the block is output in reverse block sequential order. To dereverse the decoded data, the output XDA[9:0] needs to be used as the write address to a buffer RAM.

For $NI$ odd (even half iterations), the block is output in reverse block interleaved order. To dereverse and deinterleave the block, the output XDA[9:0] is used as the write address to a buffer RAM. Note that this is only valid if one of the conditions in (10) to (12) are satisfied.

The bus ERR[7:0] is a channel error estimator output. It is the exclusive OR of XD[7:0] and the sign bits of $R0|23:0|$ and $R1|23:0|$, i.e., bits $R0|6j+5|$ and $R1|6j+5|$ for $j = 0$ to 3.

The DEC_END signal is low during decoding. At the end of decoding, DEC_END goes high. Figure 9 illustrates the decoder timing where data is output on the last half iteration. After startup, the maximum number of clock cycles for decoding is $(NI+1)(K/8+2L+12)$.

During the last half iteration the decoded and error data are stored into the interleaver memory. This occurs correctly for all half iterations, unlike the XD output while DEC_END = 0. Once decoding has been completed, the input XDE can be used to sequentially clock the decoded and error data from from the interleaver memory (regardless of the number of iterations). XDE is disabled while the decoder is iterating. Figure 10 shows the decoder timing when XDE is used.

The early stopping algorithm uses the magnitude of the extrinsic information to determine when to stop. As the decoder iterates, the magnitudes generally increases in value as the decoder becomes more confident in its decision. By comparing the smallest magnitude of a block with threshold $ZTH$, we can decide when to stop. If the smallest magnitude is greater than $ZTH$, i.e., not equal or less than $ZTH$, the decoder will stop iterating if early stopping has been enabled.

Since the last half iteration is used to store the decoded data into the interleaver memory, the decoder performs an extra half iteration once the threshold has been exceeded.

Increasing $ZTH$ will increase the average number of iterations and decrease the BER. Decreasing $ZTH$ will decrease the average number of iterations and increase the BER. In general, higher values of SNR will decrease the number of iterations. A value of $ZTH = 23$ was found to give a good trade off between the average number of iterations and BER performance.

For high SNR operation early stopping can lead to significantly reduced power consumption,
since most blocks will be decoded in one or two iterations.

**LXD Output**

The output \( \text{LXD}[15:0] \) is an estimate of the logarithm of the probability of the four decoded symbol outputs (four bits for each symbol). In the probability domain, this value ranges from 0.25 to 1 as there are four symbols. In the log domain, this corresponds to a range from 0 to 14.

The LXD output is formed by taking the \( \text{min}^* \) operation (equivalent to the \( \text{max}^* \) operation, but in the minus log domain) of the log–likelihood ratios of each of the four symbols, where the most likely symbol is used as the reference symbol. A fixed look–up table for the correction term in the \( \text{min}^* \) operation is used to allow operation at high signal to noise ratio (otherwise, due to finite quantisation, all LXD values would equal zero).

The LXD output can be used to determine the reliability of a decoded output by summation of the \( K/2 \) LXD values. Thus, one can compare the reliability of the decoded data for different conditions, choosing the decoded data that has the highest summation of LXD values.

**Simulation Software**

Free software for simulating the PCD04D4 turbo decoder in additive white Gaussian noise (AWGN) or with external data is available by sending an email to info@sworld.com.au with “pcd04d4sim request” in the subject header. The software uses an exact functional simulation of
the PCD04D4 turbo decoder, including all quantisation and limiting effects.

After unzipping pcd04d4sim.zip, there should be pcd04d4sim.exe and code.txt. The file code.txt contains the parameters for running pcd04d4sim. These parameters are

kt  No. of data bits (1)
nt  No. of coded bits (2 or 3)
m  Encoder memory (2 to 4)
g0  1st divisor polynomial of CC in octal
g1  2nd divisor polynomial of CC
g2  1st numerator polynomial of CC
g3  2nd numerator polynomial of CC
q  Number of quantisation bits (1 to 6)

EbbNom  Minimum Eb/N0 (in dB)
EbbNomax  Maximum Eb/N0 (in dB)
EbbNo  Eb/N0 increment (in dB)

optC  Input scaling parameter (normally 0.5)

ferrmax  Number of frame errors to count
Pfmin  Minimum frame error rate (FER)
Pbmin  Minimum bit error rate (BER)

NI  Number of half iterations–1 (0 to 63)
SLD  MAP decoder delay select (0 or 1)
LIMZ  Extrinsic information limit (1 to 193)
SCLZ  Extrinsic information scale (1 to 32)
M  Stopping mode (0 to 4)

ZTH  Extrinsic info. threshold (0 to 255)
KS  Data length select (−1 to 22, 32 to 39)
K  Block length (40 to 8128)
P0  1st interleave parameter (odd number)
P1  2nd input interleave parameter
P2  3rd input interleave parameter
P3  4th input interleave parameter
Q0  1st DVB–RCS2 interleave parameter
Q1  2nd DVB–RCS2 interleave parameter
Q2  3rd DVB–RCS2 interleave parameter
Q3  4th DVB–RCS2 interleave parameter

LOGMAP  Log–MAP decoding (MODE0, 0 or 1)

enter_C  Enter external C (y or n)
C  C (0 to 11)

state  State file (0 to 2)
s1  Seed 1 (1 to 2147483562)
s2  Seed 2 (1 to 2147483398)

out_screen  Output data to screen (y or n)
read_x  Use external information data (y or n)
read_r  Use external received data (y or n)
out_dir  Output directory
in_dir  Input directory

The parameter optC is used to determine the “optimum” values of A and C. The “optimum” value of A is

$$A = \frac{\text{optC}(2^{q-1} - 1)}{\text{mag}(\sigma)}$$  (15)

where $\sigma^2$ is the normalised noise variance given by (2) and mag($\sigma$) is the normalising magnitude resulting from an auto–gain control (AGC) circuit.

We have

$$\text{mag}(\sigma) = \sigma \sqrt{\frac{2}{\pi}} \exp\left(\frac{-1}{2\sigma^2}\right) + 1 - 2Q\left(\frac{1}{\sigma}\right)$$  (16)

where $Q(x)$ is the error function given by

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} \exp\left(-\frac{t^2}{2}\right)dt.$$  (17)

Although mag($\sigma$) is a complicated function, for high signal to ratio (SNR), mag($\sigma$) = 1. For low SNR, $\text{mag}(\sigma) = \sigma \sqrt{\frac{2}{\pi}} = 0.798\sigma$. That is, an AGC circuit for high SNR has an amplitude close to the real amplitude of the received signal. At lower SNR, the noise increases the estimated amplitude, since an AGC circuit averages the received signal amplitude.

For the "optimum" A, we round down the value of C given by (1) to the nearest integer. If LOGMAP = MODE0 = 0 then C is forced to 0. For LOGMAP = 1, if C is greater than 11, C is limited to 11. If C = 1, C is rounded down to 0. If C = 2 or 3, C is rounded up to 4. An external value of C can be input by setting enter_C to y.

For KS > 0, internal interleaver parameters as specified by the standard are used. For KS = 0, the parameters $P = P0$ and $P(1) = P1$ to $P(3) = P3$ are used. For KS < 0, $P = P0$ and $Q0 = Q0$ to $Q3 = Q3$ are used. The software then calculates $P(1)$ to $P(3)$ for use by the interleaver.

The simulation will increase $E_b/N_0$ (in dB) in $E_b/N_0$ increments from $E_b/N_0$ until $E_b/N_0$ is reached or the frame error rate (FER) is below or equal to Pfmin or the bit error rate (BER) is below or equal to Pbmin. Each simulation point continues until the number of frame errors is equal to ferrmax. If ferrmax = 0, then only one frame is simulated.

An optional Genie aided stopping mode can be selected by setting M = 4. This will stop the decoder from further iterations when the Genie has detected there are no errors compared to the transmitted data. This allows a lower performance bound to be simulated, allowing fast simulations for various configurations at low bit error rates. This option is not available in the decoder core.

When the simulation is finished the output is given in, for example, file k472.dat, where $K = 472$. The first line gives the $E_b/N_0$ ($E_b/N_0$ in dB) in $E_b/N_0$ increments from $E_b/N_0$ until $E_b/N_0$ is reached or the frame error rate (FER) is below or equal to Pfmin or the bit error rate (BER) is below or equal to Pbmin. Each simulation point continues until the number of frame errors is equal to ferrmax. If ferrmax = 0, then only one frame is simulated.
The average number of iterations (na), the average BER (\(P_b\)) and the average FER (\(P_f\)). Following this, the number of iterations, \(na\), \(berr\), \(ferr\), \(P_b\), and \(P_f\) are given for each half iteration.

The following file was used to give the simulation results shown in Figure 11 for \(K = 472\) (KS = 4) and max–log–MAP decoding. Auto–stopping was used with up to 10 iterations. When iterating is stopped early, the nasum(2*num*na), berr and ferr results at stopping are copied for each half iteration to the maximum iteration number. Figure 12 shows the average number of iterations with \(E_b/N_0\).

<table>
<thead>
<tr>
<th>(K)</th>
<th>(E_b/N_0) (dB)</th>
<th>(optC)</th>
<th>(A)</th>
<th>(C)</th>
<th>(SCLZ)</th>
<th>(BER)</th>
</tr>
</thead>
<tbody>
<tr>
<td>112</td>
<td>1.40</td>
<td>0.50</td>
<td>14.08</td>
<td>5</td>
<td>31</td>
<td>10^{-2}</td>
</tr>
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<td>0.45</td>
<td>12.54</td>
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<td>31</td>
<td>2.43</td>
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<tr>
<td>472</td>
<td>0.97</td>
<td>0.45</td>
<td>12.47</td>
<td>5</td>
<td>32</td>
<td>2.96</td>
</tr>
<tr>
<td>680</td>
<td>0.90</td>
<td>0.44</td>
<td>12.16</td>
<td>4</td>
<td>32</td>
<td>2.38</td>
</tr>
</tbody>
</table>

The \(state\) input can be used to continue the simulation after the simulation has been stopped, e.g., by the program being closed or your computer crashing. For normal simulations, \(state = 0\). While the program is running, the simulation state is alternatively written into state1.dat and state2.dat. Two state files are used in case the program stops while writing data into one file. To continue the simulation after the program is stopped follow these instructions:

1) Copy the state files state1.dat and state2.dat. This ensures you can restart the program if a mistake is made in configuring code.txt.
2) Examine the state files and choose one that isn’t corrupted.
3) Change the state parameter to 1 if state1.dat is used or 2 if state2.dat is used.
4) Restart the simulation. The output will be appended to the existing k(K).dat file.
5) After the simulation has been completed, make sure that state is changed back to 0.

The software can also be used to encode and decode external data. To encode a block x_(K).dat in the directory given by in_dir, set read_x to y, e.g., x_472.dat in directory input (each line contains one bit of data). The data is input in the order A(0) B(0) ... A(K/2–1) B(K/2–1).

The encoded stream y_(K).dat will be output to the directory given by out_dir, e.g., y_472.dat to directory output. The encoded data is output in the order A(0) B(0) ... A(K/2–1) B(K/2–1) Y1(K/2–1) Y2(K/2–1) W1(0) W2(0) ... W1(K/2–1) W2(K/2–1).

To decode data, place the received block of data in file r_(K).dat in directory in_dir and set read_r to y. The decoded data is output to xd_(K).dat in directory out_dir. The file r_(K).dat has in each line R[i,j], i = 0 to 3 or 5 from j = 0 to K/2–1, e.g., the first three lines of rate 1/2 data could be

```
-31  1  -25  27
-31  12  -4  9
 11  31  31  2
```

The input data is of the form

\[ R[i,j] = A^*(1-2^*Y[i,j]+N[i,j]) \]

where A is the signal amplitude, Y[i,j] is the coded bit, and N[i,j] is white Gaussian noise with zero mean and normalised variance \( \sigma^2 \). The magnitude of R[i,j] should be rounded to the nearest integer and be no greater than \( 2^{q-2} - 1 \). If read_r = y, then C is externally input via c.

## Ordering Information

SW–PCD04D4–SOS (SignOnce Site License)
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• 0.00 22 January 2015. Preliminary product specification.
• 1.00 2 February 2015. First official release. Added performance curves and parameters.
• 1.02 22 February 2015. Updated Table 1.
• 1.05 12 December 2018. Clarified XD[7:0] output.